British Mathematical Olympiad

Round 1 : Wednesday 15th January 1992

Time allowed *Three and a half hours.*

Instructions

- *Full written solutions are required. Marks awarded will depend on the clarity of your mathematical presentation. Work in rough first, and then draft your final version carefully before writing up your best attempt. Do not hand in rough work.*
- *One complete solution will gain far more credit than several unfinished attempts. It is more important to complete a small number of questions than to try all five problems.*
- *The first two questions are intended to be more straightforward than the last three.*
- *The use of rulers and compasses is allowed, but calculators are forbidden.*
- *Start each question on a fresh sheet of paper. Write on one side of the paper only. On each sheet of working write the number of the question in the top left hand corner and your name, initials and school in the top right hand corner.*
- *Complete the cover sheet provided and attach it to the front of your script, followed by the questions 1,2,3,4,5 in order.*
- *Staple all the pages neatly together in the top left hand corner.*

Do not turn over until **told to do so.**

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1. (a) Observe that the square of 20 has the same number of non-zero digits as the original number. Does there exist a two-digit number, *other than* 10, 20 or 30, whose square has the same number of non-zero digits as the original number? If you think there is one, then find it. If you claim that there is none, then you must prove your claim.

   (b) Does there exist a three-digit number *other than* 100, 200, 300 whose square has the same number of non-zero digits as the original number?

2. Let $ABCDE$ be a pentagon inscribed in a circle. Suppose that AC, BD, CE, DA and EB are parallel to DE, EA, AB, BC and CD, respectively. Does it follow that the pentagon has to be regular? Justify your claim.

3. Find four distinct positive integers whose product is divisible by the sum of every pair of them.

   Can you find a set of five or more numbers with the same property?

4. Determine the smallest value of $x^2+5y^2+8z^2$, where x, y, z are real numbers subject to the condition $yz+zx+xy=-1$. Does $x^2+5y^2+8z^2$ have a greatest value subject to the same condition? Justify your claim.

5. Let f be a function mapping the positive integers into positive integers. Suppose that $f(n+1) > f(n)$ and $f(f(n)) = 3n$ for all positive integers n. Determine $f(1992)$.